IS 8161-4 (1985): Guide for equipment reliability testing, Part 4: Procedure for determining point estimates and confidence limits from equipment determination tests [LITD 2: Reliability of Electronic and Electrical Components and Equipment]

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Indian Standard

GUIDE FOR EQUIPMENT RELIABILITY TESTING

PART 4 PROCEDURE FOR DETERMINING POINT ESTIMATES AND CONFIDENCE LIMITS FROM EQUIPMENT RELIABILITY DETERMINATION TESTS

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Indian Standard

GUIDE FOR EQUIPMENT RELIABILITY TESTING

PART 4 PROCEDURE FOR DETERMINING POINT ESTIMATES AND CONFIDENCE LIMITS FROM EQUIPMENT RELIABILITY DETERMINATION TESTS

0. FOREWORD

0.1 This Indian Standard (Part 4) was adopted by the Indian Standards Institution on 14 June 1985, after the draft finalized by the Reliability of Electronic and Electrical Components and Equipment Sectional Committee had been approved by the Electronics and Telecommunication Division Council.

0.2 This standard (Part 4) is one of the series of Indian Standards for equipment reliability testing and covers procedures for determining point estimates and confidence limits from equipment reliability determination tests. Other standards available in this series are given on page 28.

0.3 This standard is largely based on 'IEC Doc: 56 (Central Office) 92 Draft, IEC Pub 605 Equipment reliability testing: Part 4 Procedures for determining point estimates and confidence limits from equipment reliability determination tests' issued by the International Electrotechnical Commission (IEC).

0.4 This standard is one of a number of Indian Standards on reliability of electronic and electrical components and equipment. A list of standards published so far in this area is given on page 28.

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1. SCOPE

1.1 This standard (Part 4) gives recommended numerical and graphical methods for determining point estimates and confidence limits of reliability characteristics from equipment reliability determination tests.

2. TERMINOLOGY

2.0 For the purpose of this standard, the terms and definitions as given in IS : 1885 (Part 39)-1979* and IS : 7690-1975† shall apply.

3. LIST OF SYMBOLS

3.1 The following symbols are used in this standard. The 'determination point' is the point in time or number of trials at which the point estimates or confidence limits are determined.

a	= location parameter in a Weibull distribution;
b	= charateristic life (or scale parameter) in a Weibull distribution;
$f(t)$	= probability density function of time to failure;
$F(t)$	= cumulative distribution of times to failure, probability of failure within time t;
$F_p(\nu_1, \nu_2)$	= theoretical value of the F-distribution with ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator at the fractile of order p;
i	= order number of a failure based on the time to failure (t_i);
k	= shape parameter of a Weibull distribution;
m	= true mean time between failures;
$\hat{m}$	= point estimate of mean time between failures (observed value);
m_F	= true mean time to failure, mean life;
$\hat{m}_F$	= point estimate of mean time to failure (observed value;
$m(O, t_o)$	= true mean time between failures for a period of time (O, t_o);

*Electrotechnical vocabulary: Part 39 Reliability of electronic and electrical items (*first revision*).

†Mathematical guide to the terms and definitions for reliability of electronic equipment and components (or parts) used therein.

$\hat{m}$ (O, t_0) = point estimate of mean time between failures for the period of time (O, t_0) (observed value);

n = relevant number of test items or trials;

P_{50} (t_i) = 50 percent (median) ranks of t_i (*see* Table 1);

r = total number of relevant failures on which the determination test is based;

R = true success ratio;

$\hat{R}$ = point estimate of success ratio (observed value);

$R(t)=1-F(t)$ = probability of success;

t = time (or equivalent, such as distance, cycles or other quantites as appropriate);

t^*_m = predetermined test time for the mth item;

t_i = relevant test time of magnitude order number i, recorded between the point of time the corresponding item was put on test and the point of time of failure of the item;

$t_p(\nu)$ = theoretical value of the Student t distribution with ν degrees of freedom at the fractile of order p;

T^* = accumulated relevant test time up to the determination point when this does not coincide with a failure;

T_r = relevant test time accumulated for all items up to the determination test point when this coincides with a failure;

λ = true constant failure rate;

$\hat{\lambda}$ = point estimate of failure rate (observed value);

σ = true standard deviation in normal distribution;

$\hat{\sigma}$ = point estimate of the standard deviation; and

$X^2_p(\nu)$ = theoretical value of the X^2 distribution with ν degrees freedom at the fractile of order p.

TABLE 1 MEDIAN RANKS (P 50 IN PERCENT)

(*Clauses* 3.1, 6.3.1, 7.1.1, 7.1.2 *and* 7.2.2.1)

ORDER No (i)	SAMPLE SIZE (n) 1	2	3	4	5	6	7	8	9	10
1	50·0	29·2	20·6	15·9	12·9	10·9	9·4	8·3	7·4	6·6
2		70·7	50·0	38·5	31·3	26·6	22·8	20·1	17·9	16·2
3			79·3	61·4	50·0	42·1	36·4	32·0	28·6	25·8
4				84·0	68·6	57·8	50·0	44·0	39·3	35·5
5					87·0	73·5	63·5	55·9	50·0	45·1
6						89·0	77·1	67·9	60·6	54·8
7							90·5	79·8	71·3	64·4
8								91·7	82.0	74·1
9									92·5	83·7
10										93·3

ORDER No. (i)	SAMPLE SIZE (n) 11	12	13	14	15	16	17	18	19	20
1	6·1	5·6	5·1	4·8	4·5	4·2	3·9	3·7	3·5	3·4
2	14·7	13·5	12·5	11·7	10·9	10·2	9·6	9·1	8·6	8·2
3	23·5	21·6	20·0	18·6	17·4	16·3	15·4	14·5	13·8	13·1
4	32·3	29·7	27·5	25·6	23·9	22·4	21·1	20·0	18·9	18·0
5	41·1	37·8	35·0	32·5	30·4	28·5	26·9	25·4	24·1	22·9
6	50·0	45·9	42·5	39·5	36·9	34·7	32·7	30·9	29·3	27·8
7	58·8	54·0	50·0	46·5	43·4	40·8	38·4	36.3	34·4	32·7
8	67·6	62·1	57·4	53·4	50·0	46·9	44·2	41·8	39·6	37·7
9	76·4	70·2	64·9	60·4	56·5	53·0	50·0	47·2	44.8	42·6
10	85·2	78·3	72·4	67·4	63·0	59·1	55·7	52·7	50·0	47·5
11	93·8	86·4	79·9	74·3	69·5	65·2	61·5	58·1	55·1	52·4
12		94.3	87·4	81·3	76·0	71·4	67·2	63·6	60·3	57·3
13			94·8	88·2	82·5	77·5	73·0	69·0	65·5	62·2
14				95·1	89·0	83·6	78·8	74·5	70·6	67·2
15					95·4	89·7	84·4	79·9	75·8	72·1
16						95·7	90·3	85·4	81·0	77·0
17							96·0	90·8	86·1	81·9
18								96·0	91·3	86·8
19									96·4	91·7
20										96·5

NOTE — For higher values of n and i use formula:

$$P_{50} = 100 \left(\frac{i - 0{\cdot}3}{n + 0{\cdot}4} \right)$$

4. APPLICABILITY

4.1 The methods of calculation in this standard may be applied at any time or after any number of trials of the reliability testing. The more information that is available when the estimates and limits are determined, the higher is the precision obtained. Existing data from earlier tests or field observations may be acceptable, provided the data are sufficiently complete, well establised and applicable to the situation. The standard is not applicable to the pooling of data from different sources, for example, original data from tests under different conditions.

4.2 Wherever 'time' is used in this standard, this variable may be replaced by distance, cycles or other quanties as may be appropriate.

4.3 If a constant failure rate, a mean time to/between failures, or a mean failure rate for a given period of time is to be estimated from the reliability determination test, the methods in 6 or 7 should be used. These methods are applicable for tests based on time. The method to be used depends on what assumption can be made regarding the failure rate dependence on time and the statistical distribution of time to/between failures.

4.4 In case of an assumption of a constant failure rate characterising an exponential distribution of times to/between failures, the method given in 6 is to be used. The test items may be repaired and put back on test or not repaired. The graphical method (6.3) is, however, not applicable after the first failure of repaired test items. In case of an assumption of a non-constant failure rate described by a Weibull distribution or a normal (Gaussian) distribution of times to failure, the methods given in 7 should be used. These methods are not applicable after the first failure of repaired test items.

4.5 If the success ratio is to be estimated from a reliability determination test, the methods in 8 are to be used. These methods are based on the number of test items or trials, which are classified as either failed or non-failed. The method to be used depends on the number of failures. If the number of failures is greater than 9 the normal distribution may be used, in other cases the binominal distribution shall be used.

5. FEATURES OF THE POINT ESTIMATES AND CONFIDENCE LIMITS

5.1 A point estimate is a single numerical value to represent the unknown true value of a statistical parameter, such as the failure rate. Generally, the point estimate considered here is the 'observed' value [as defined in IS : 1885 (Part 39)-1979* and IS : 7690-1975†].

5.2 The confidence limits define a confidence interval around the point estimate where the interval includes the true value of the parameter being estimated with a certain probability, the confidence level.

5.3 The confidence interval will be narrower when more information is available. The information used is the accumulated relevant test time and the number of failures for time based tests, and the number of test

*Electrotechnical vocabulary: Part 39 Reliability of electronic and electrical items (*first revision*).

†Mathematical guide to the terms and definitions for reliability of electronic equipment and components (or parts) used therein.

items or trials and the number of failures for tests based on items or trials.

5.4 The confidence interval may be one-sided or two-sided. In case of a one-sided confidence interval, a lower or an upper confidence limit is given for the characteristic. In case of a two-sided confidence interval, both a lower and an upper limit are given.

5.5 The preferred confidence level is here chosen to be 90 percent. The confidence intervals according to this recommendation therfore include the true values of the characteristics with 90 percent probability. If other confidence levels are desired, the formulae in this standard may be used with appropriate values of the test statistics taken from statistical tables.

6. CONSTANT FAILURE RATE

6.0 If the true failure rate is assumed to be constant, the numerical and graphical methods in **6.1**, **6.2** and **6.3** may be used for estimating failure rate and mean time between failures (for repaired items) or mean time to failure (for non-repaired items). The methods are applicable both to repair and non-repair situations. These estimates are based on the total number of relevant failures and the accumulated relevant test time up to the determination point.

6.0.1 The validity of a constant failure rate assumption should preferably be tested in accordance with IS : 8161 (Part 6)-1983* before point estimates and confidence limits are calculated.

For application to mean time between failures, M.T.B.F. $= m$.

For application to mean time to (first) failure, m_F or M.T.T.F.:

$$m_F = \text{M.T.T.F.} = \text{m}.$$

For application to failure rate $\lambda = \dfrac{1}{m}$

6.1 Time Terminated Tests

6.1.0 The accumulated relevant test time up to the determination point, T^*, may be determined in accordance with Appendix A. The same formulae are used for test with and without replacement or repair. The total number of relevant failure, r, up to the determination point is counted.

*Guide for equipment reliability testing : Part 6 Tests for validity of a constant failure rate assumption.

6.1.1 *Point Estimates*

The point estimate (observed value) of the failure rate is:

$$\hat{\lambda} = \frac{r}{T^*}$$

The point estimate (observed value) of the mean time to or between failures is:

$$\hat{m} = \frac{T^*}{r}$$

If no failures are observed up to the termination point, that is, $r = 0$, the following point estimate of the failure rate is recommended:

$$\hat{\lambda} = 1/3T^*$$

Too much reliance should not be placed on estimates based upon zero or any small number of failures.

Care should be taken in planning the tests such that T^* will be sufficiently large in relation to the expected mean time to failure.

NOTE — This recommendation is based upon an investigation by E.L. Welker and M. Lipow 'Estimating the exponential failure rate from data with no failure events' in Proceedings of 1974 Annual Reliability and Maintainability Symposium, pages 420-427.

6.1.2 *Confidence Intervals*

6.1.2.1 The confidence limits for the true failure rate, λ, with the confidence level of 90 percent are shown below (*see* Table 2 for values of the X^2 distribution).

a) One-sided confidence interval, upper limit:

$$\lambda < \hat{\lambda}\,\frac{\chi^2_{0\cdot 90}\,(2r+2)}{2r} \text{ or } \lambda < \frac{\chi^2_{0\cdot 90}\,(2r+2)}{2\,T^*}$$

b) Two-sided confidence interval:

$$\hat{\lambda}\,\frac{\chi^2_{0\cdot 05}\,(2r)}{2r} < \lambda < \hat{\lambda}\,\frac{\chi^2_{\cdot 95}\,(2r+2)}{2r}$$

or

$$\frac{\chi^2_{0\cdot 05}\,(2r)}{2T^*} < \lambda < \frac{\chi^2_{0\cdot 95}\,(2r+2)}{2T^*}$$

If no failures are observed only the one-sided confidence limit with an upper limit can be defined.

TABLE 2 CHI-SQUARE DISTRIBUTION FRACTILES

(*Clauses* 6.1.2.1, 6.1.2.2, 6.2.2.1 *and* 6.2.2.2)

Degrees of Freedom ν	$\chi^2_{0\cdot05}(\nu)$	$\chi^2_{0\cdot90}(\nu)$	$\chi^2_{0\cdot95}(\nu)$
2	·103	4·605	5·991
4	·711	7·779	9·488
6	1·635	10·65	12·59
8	2·733	13·36	15·51
10	3·94	15·98	18·31
12	5·226	18·55	21·03
14	6·571	21·06	23·69
16	7·962	23·54	26·3
18	9·39	25·99	28·87
20	10·85	28·41	31·41
22	12·34	30·81	33·92
24	13·85	33·2	36·42
26	15·38	35·56	38·89
28	16·92	37·92	41·34
30	18·49	40·26	43·77
32	20·09	42·57	46·17
34	21·7	44·88	48·57
36	23·3	47·19	50·96
38	24·91	49·5	53·36
40	26·51	51·81	55·76
42	28·16	54·08	58·11
50	34·76	63·17	67·51
52	36·45	65·42	69·82
60	43·19	74·4	79·08
62	44·9	76·63	81·37
70	51·74	85·53	90·53
72	53·47	87·74	92·8
80	60·39	96·58	101·88
82	62·14	98·78	104·13
90	69·13	107·57	113·15
92	70·89	109·76	115·39
100	77·93	118·5	124·34
102	79·74	120·65	126·53
110	86·96	129·25	135·3
112	88·77	131·4	137·5
120	96	140	146.27
122	97·81	142·15	148·46
200	168·28	226·02	233·99
$z=$	$-1\cdot64$	$+1\cdot29$	$+1\cdot64$

NOTE 1 — Linear interpolation of intermediate values is sufficiently accurate. Values for $\nu=2r+2$ for various integer r values are included.

NOTE 2 — For higher values, use:

$\chi_p(\nu)=[(z+\sqrt{2\nu-1})^2]/2$ where z is the corresponding percentage of the standard normal distribution given at the foot of each column.

6.1.2.2 The confidence limits for the true mean time to or between failures m, with a confidence level of 90 percent are shown below (*see* Table 2 for values of the χ^2 distribution).

a) One-sided confidence interval, lower limit:

$$m > \hat{m} \frac{2r}{\chi^2_{0.90} \ (2r + 2)} \text{ or } m > \frac{2T^*}{\chi^2_{0.90} \ (2r + 2)}$$

b) Two-sided confidence interval:

$$\hat{m} \frac{2r}{\chi^2_{0.95} \ (2r + 2)} < m < \hat{m} \frac{2r}{\chi^2_{0.05} \ (2r)}$$

or

$$\frac{2T^*}{\chi^2_{0.95} \ (2r + 2)} < m < \frac{2T^*}{\chi^2_{0.05} \ (2r)}$$

If no failures are observed, only the lower limit can be defined.

The 90 percent confidence limits as a function of the number of failures, r, are shown in Fig. 1 and Table 3. The limits are here expressed by the appropriate multiplier times the relevent point estimate $\hat{\lambda}$ and $\hat{m}$.

NOTE — The multipliers may be used in the planning of life tests by finding out the approximate number of failures for a given precision of the estimate. The required accumulated test time, T^*, for the required number of failures is approximately $T^* = \gamma/\lambda$, where the failure rate λ has to be assumed from earlier experience.

6.2 Failure Terminated Tests — The accumulated relevent test time up to the determination point, T_r, is determined in accordance with Appendix A. The same formulae are used for tests with and without replacement or repair.

6.2.1 *Point Estimates* — The point estimate (observed value) of the failure rate is:

$$\hat{\lambda} = \frac{r}{Tr}$$

The point estimate (observed value) of the mean time to or between failures is:

$$m = \frac{Tr}{r}$$

Too much reliance should not be placed on estimates based upon a small number of failures.

Care should be taken in planning the tests such that T^* will be sufficiently large in relation to the expected mean time to failure.

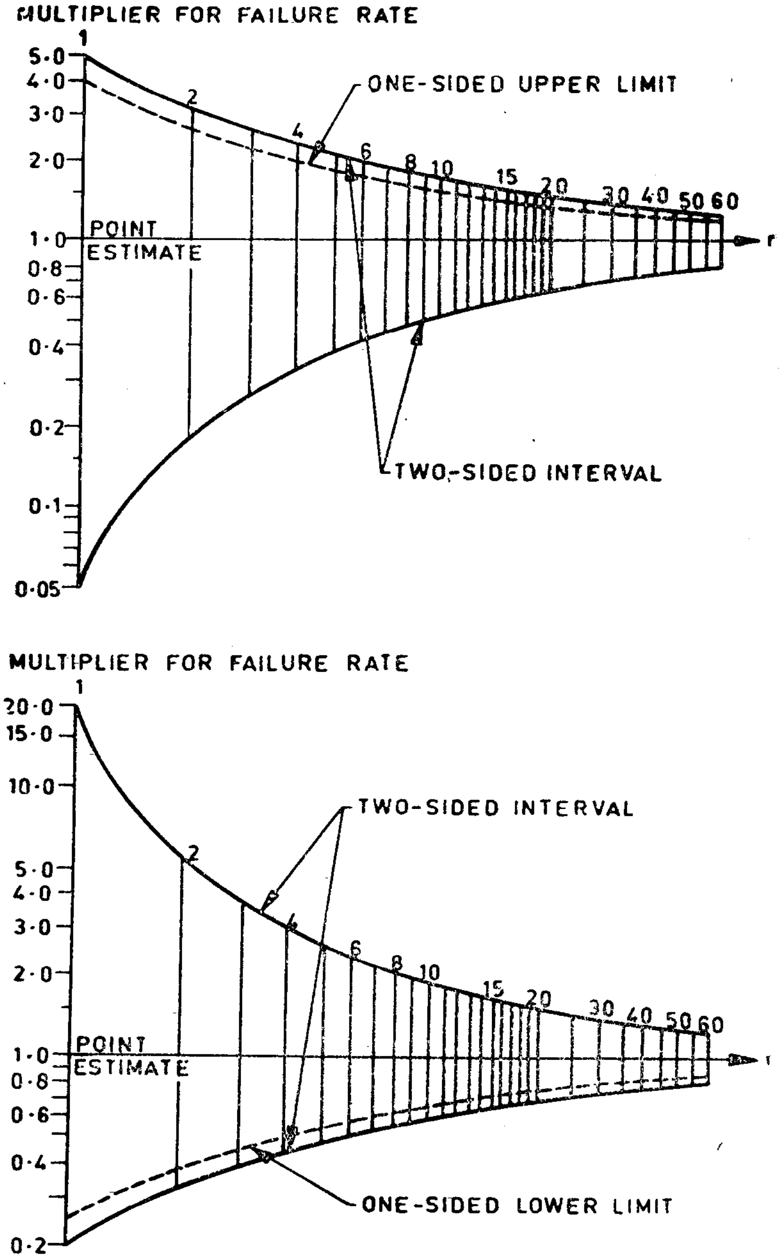


FIG. 1 THE 90 PERCENT CONFIDENCE LIMITS AS A FUNCTION OF NUMBER OF FAILURES FOR TIME—TERMINATED TESTS

TABLE 3 MULTIPLIERS TO OBTAIN CONFIDENCE LIMITS

(*Clause* 6.1.2.2)

R	Multipliers for MTTF/MTBF			Multipliers for Failure Rate		
	M (5 percent)	M (90 percent)	M (95 percent)	M (5 percent)	M (90 percent)	M (95 percent)
1	19·417	·257	·21	·051	3·889	4·743
2	5·625	·375	·317	·177	2 662	3·147
3	3·669	·449	·386	·272	2·226	2·584
4	2·927	·5	·436	·341	1·997	2·288
5	2·538	·539	·475	·394	1·854	2·103
6	2·296	·569	·506	·435	1·755	1·974
7	2·13	·594	·532	·469	1·681	1·878
8	2·009	·615	·554	·497	1·624	1·804
9	1·916	·633	·573	·521	1·578	1·745
10	1·843	·649	·589	·542	1·54	1·695
11	1·782	·662	·604	·56	1·509	1·655
12	1·732	·674	·617	·577	1·481	1·62
13	1·69	·685	·628	·591	1·458	1·59
14	1·654	·695	·639	·604	1·437	1·563
15	1·622	·704	·649	·616	1·419	1·538
16	1·592	·713	·658	·627	1·402	1·517
17	1 566	·72	·667	·638	1·387	1·498
18	1·545	·727	·674	·647	1·375	1·482
19	1·525	·733	·681	·655	1·363	1·467
20	1·508	·739	·688	·662	1·351	1·452
25	1·438	·764	·716	·695	1·308	1·396
30	1·389	·782	·737	·719	1 277	1·356
35	1·352	·797	·754	·739	1·253	1·325
40	1·324	·809	·768	·754	1·234	1·301
45	1·301	·819	·779	·768	1·219	1 282
50	1·283	·828	·79	·779	1·206	1 265
55	1·264	·837	·8	·79	1·194	1·25
60	1·249	·844	·808	·8	1·184	1·237

6.2.2 *Confidence Intervals*

6.2.2.1 The Confidence limits for the true failure rate with a confidence level of 90 percent are shown below (*See* Table 2 for values of the χ^2 distribution):

a) One-sided confidence interval, upper limit:

$$\lambda < \hat{\lambda} \frac{\chi^2_{0\cdot 90} (2r)}{2r} \quad \text{or} \quad \lambda < \frac{\chi^2_{0\cdot 90} (2r)}{2Tr}$$

b) Two-sided confidence interval:

$$\hat{\lambda} \frac{\chi^2_{0\cdot 05} (2r)}{2r} < \lambda < \hat{\lambda} \frac{\chi^2_{0\cdot 95} (2r)}{2Tr}$$

or

$$\frac{\chi^2_{0\cdot 05} (2r)}{2Tr} < \lambda < \frac{\chi^2_{0\cdot 95} (2r)}{2Tr}$$

6.2.2.2 The confidence limits for the true mean time to or between failures with a confidence level of 90 percent are shown below, (*see* Table 2 for values of the χ^2 distribution).

a) One-sided confidence interval, lower limit:

$$m > \hat{m} \frac{2r}{\chi^2_{0.90}(2r)} \text{ or } m > \frac{2Tr}{\chi^2_{0.90}(2r)}$$

b) Two-sided confidence interval:

$$\hat{m} \frac{2r}{\chi^2_{0.95}(2r)} < m < \hat{m} \frac{2r}{\chi^2_{0.05}(2r)}$$

or

$$\frac{2Tr}{\chi^2_{0.95}(2r)} < m < \frac{2Tr}{\chi^2_{0.05}(2r)}$$

6.3 Graphical Method

6.3.0 A method is presented for estimating failure rate and mean time to first failure or mean time between failures, based on the semi-log paper. The tests from which the observations are taken need not go on until all test items have failed.

The semi-log paper should be used only for times to first failure and for not less than 4 failures. The method will give point estimates and may also give an indication of deviations from a constant failure rate.

6.3.1 A number of items, n, are tested and r failures observed. For each failed item, the relevant test time t_i is recorded.

The times t_i are ordered in magnitude:

$$t_1 < t_2 < t_3 \ldots\ldots < tr$$

On a semi-log paper (*see* Fig. 2), the t_i values are plotted along the linear scale and the reciprocal of one minus the fractional median rank, $P_{50}(t_i)/100$ along the logarithmic scale. Values of median ranks may be found from Table 1. If a constant failure rate assumption holds, the plotted points will fit well to a straight line passing through the point, $t_i = 0$, ratio = 1. In drawing the line, the central points should dominate the determination of the slope. This being so, then the estimate of the mean time to failure equals the t value on the time axis that corresponds with the ratio 2·72 on the vertical axis. The estimate of the failure rate is then the inverse of the t value.

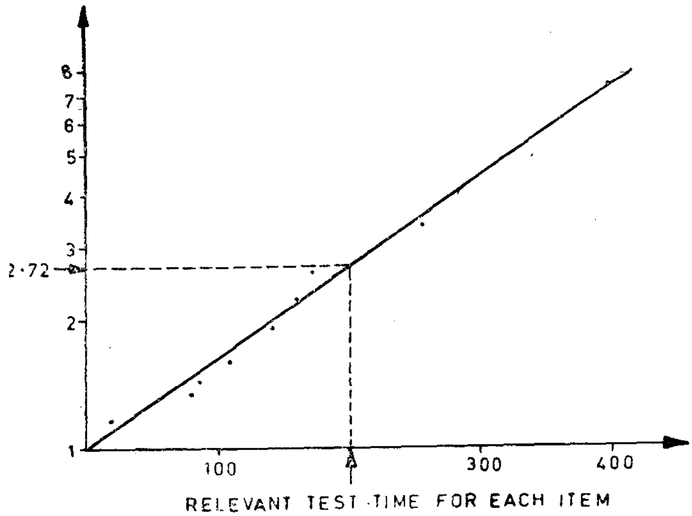


FIG. 2 EXAMPLE OF USE OF SEMI-LOG PAPER

$$\frac{1}{1 - P_{50}\,(t_i)/100}$$

$$\hat{m} = \frac{1}{\hat{A}}$$

If the plotted points cannot be fitted by a straight line, the failure rate may not be constant. In such a case, other methods as described in 7 may apply.

6.3.2 *Example*

6.3.2.1 Figure 2 has been drawn with data collected by putting 10 items in reliability test which was terminated when 8th failure was observed. The observation and calculations are given below:

Failure Order	*Test Time* (t_i) *and* i^{th} *Failure* (hours)	$p_{50}\,(t_i)$* percent	$\frac{1}{1-\frac{p_{50}\,(t_i)}{100}}$
1	29	6·6	1·07
2	29·9	16·2	1·19
3	30·6	25·8	1·38
4	32·4	35·5	1·55
5	33·0	45·1	1·82
6	35·3	54·8	2·21
7	36·1	64·4	2·81
8	40·1	74·1	3·86

6.3.2.2 The above data is plotted on semi-log paper in Fig. 2 as described in **6.3.1** and observed value for estimated failure rate at the ratio value of 2·72 is equal to $\frac{1}{36\cdot5}$ h = 0·027 4 failures/hour.

*Obtained from Table 5.

7. NON-CONSTANT FAILURE RATE

7.0 If the assumption of a constant failure rate is not valid and the distribution of times to failure follow a Weibull or a normal distribution, the methods given below are applicable. The validity of any of these distribution assumptions should preferably be tested in accordance with IS : 8161 (Part 8)* or using the probability papers described in **7.1** and **7.2**.

7.0.1 The life history of the test items shall be similar when they are put on test. The methods of evaluation are applicable only to the times to first failure of each item. The test does not, however, need to be continued until all items have failed, except for the method in **7.2.1.1**.

7.0.2 For each failed test item, the relevant test time, t_i, is determined. The time t_i is defined as the relevant test time between the point of time when the item i was put on test, $t = 0$, and the point of failure, $t = t_i$. The times t_i are ordered in magnitude

$$t_1 < t_2 < t_3 < \ldots\ldots < t_r$$

Using these times and a Weibull probability paper or a normal probability paper it is possible to estimate the parameters of the distribution and at the same time obtain an indication of how well the times are fitted to the assumed distribution.

7.1 Weibull Distribution

7.1.0 The cumulative distribution function for the Weibull distribution is:

$$F(t) = 1 - e^{\left(\frac{t-a}{b}\right)k}; \qquad > a, b > o, k > o$$

7.1.1 A graphical method for point estimates is presented below. In this method, the Weibull probability paper is used. This paper is so designed that the cumulative distribution function of times to failure which follow a Weibull distribution, will become a straight line. See Fig. 3 which is an example of such a paper and how it can be used.

If the time scale on the abscissa is not directly applicable, it can be transformed by a scale factor 10^c, where c is an interger.

The observed t_i values are plotted at the corresponding median rank values (P_{50}) (t_i) on the $F(t)$ scale. Median rank values can be found in Table 1.

*Guide for equipment reliability testing: Part 8 Tests for validity of a non-constant failure rate assumption (*under consideration*).

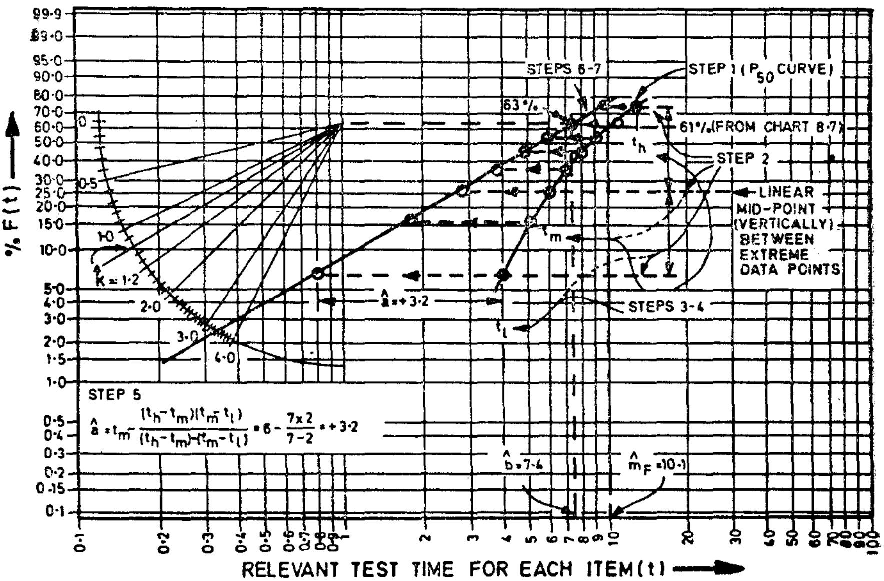


FIG. 3 EXAMPLE OF USE OF WEIBULL PROBABILITY PAPER

7.1.2 The example of Fig. 3 assumes a test sample $n = 10$ when the test is terminated after $r = 8$ failures. Therefore, there are only 8 plotted points and in the table of median ranks (Table 1) the column for $n = 10$ is used up to and including $i = 8$ only.

7.1.3 If a Weibull distribution with $a = 0$ holds then the plotted points can be fitted will by a straight line. If the points follow a concave curve seen from the abscissa, the location parameter a has a positive value. If the curve is convex then a is negative. For a curve the following procedure may be used:

Step 1: Plot curved data as described above and illustrated in Fig. 3.

Step 2: Draw two parallel horizontal lines through the extreme failure points, and a third horizontal parallel line at the linear mid-point.

Step 3: Draw three lines perpendicular to the t-axis from the points of intersection of the three horizontal lines and the curve.

Step 4: Identify the lowest, middle, and highest t value as t_l, t_m and t_h respectively.

Step 5: Use the following formula to calculate:

$$\hat{a} = t_m - \frac{(t_h - t_m) (t_m - t_l)}{(t_h - t_m) (t_m - t_l)}$$

Step 6: Subtract $\hat{a}$ (including sigh of $\hat{a}$) from each t_i value.

Step 7: Replot the data; the points should now approximate a straight line, a shifted Weibull distribution of failures.

7.1.3.1 The data plotted in Fig. 3 was collected on 10 items kept on reliability test and the test was terminated at 8th failure. The observed failures and calculations as described above are given below:

Failure Order	*Test Time, at i*th *Failure* (hours)	*Medium Rank Value* percent*
1	400	6·6
2	510	16·2
3	620	25·8
4	720	35·5
5	820	45·1
6	920	54·8
7	1 050	64·4
8	1 300	74·1

7.1.3.2 From above data, t_l = 400, t_h = 1 300 and t_m = 620 and hence a = 294·8 (≈ 300).

7.1.4 To estimate the shape parameter, k, a line is drawn through the centre of the circular scale parallel to the fitted straight line. The value of k is then read off the circular scale at the intersection.

The estimate of the characteristic life or scale parameter, b, is equal to the t value of the intersection of the fitted straight line and dashed line on the vertical axis at 6396.

7.1.5 To estimate mean life m_F refer to Fig. 4. This gives for different k values the value of $F (t)$ when $t = mF$. Then by reference to the ordinate scale of $F (t)$ in Fig. 3, m_F can be read off from the *original* plot (not the straight line derived from a) at the abscissa scale. The original plot may also be used to estimate the fraction $F (t)$, of the population having times to failure less than a given t value.

7.1.6 Other available types of graph paper, while basically similar, may use different methods to estimate the parameters as instructed.

*Taken from Table 5.

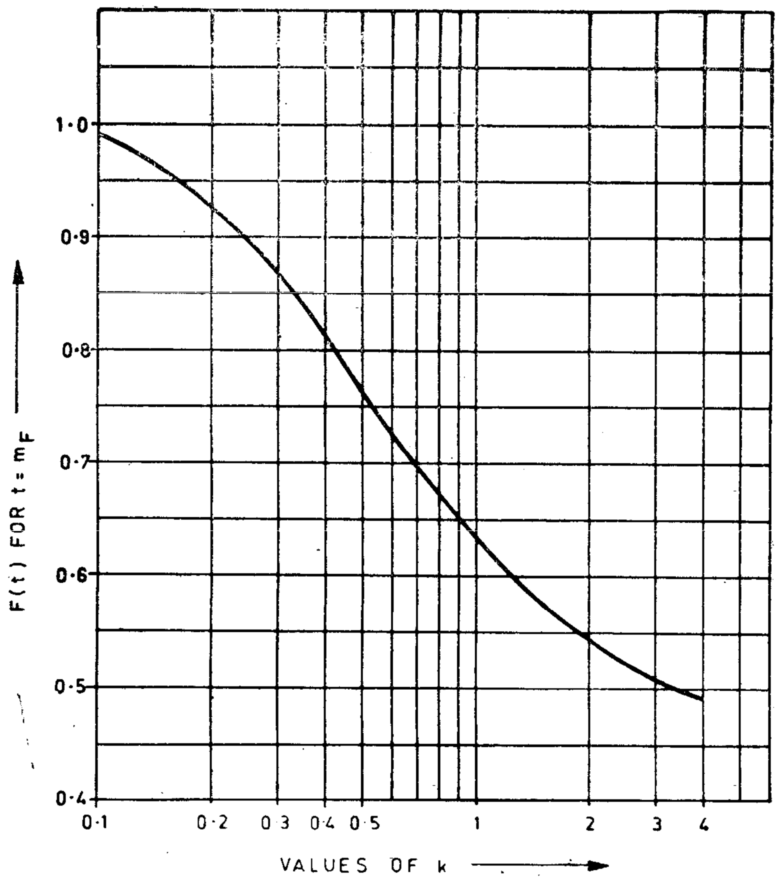


FIG. 4

7.2 Normal Distribution

7.2.0 The probability density function of the normal distribution is:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}}\, e - \frac{(t - m_f)^2}{2\sigma^2}$$

This distribution is useful for times to failure, provided that $m_F > 3\sigma$.

7.2.1 *Numerical Method*

7.2.1.1. *Point estimates* — The numerical estimates of the mean time to failure and standard deviation presented here are applicable only when all the n test items are tested until they fail.

a) The point estimate of the mean time to failure is:

$$\hat{m}_F = \frac{1}{n} \sum_{i=1}^{n} t_i$$

b) The point estimate of the standard deviation is:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (t_i - \hat{m}_F)^2}{n - 1}} = \sqrt{\frac{n \sum_{i=1}^{n} t_i^3 - (\sum_{i=1}^{n} t_i)^2}{n (n - 1)}}$$

7.2.1.2 *Confidence intervals* — The confidence limits for the true mean time to failure, m_F, with a confidence level of 90 percent are shown below [*see* Table 4 for values of the Student's distribution $t_p (\nu)$].

TABLE 4 STUDENT *t* DISTRIBUTION

Degrees of Freedom ν	$t_{0 \cdot 90}$ (ν)	$t_{0 \cdot 95}$ (ν)	Degrees of Freedom ν	$t_{0 \cdot 90}$ (ν)	$t_{0 \cdot 95}$ (ν)
2	1·89	2·92	14	1·34	1·76
3	1·64	2·35	16	1·34	1 75
4	1·53	2·13	18	1·33	1·73
5	1·48	2·02	20	1·33	1.72
6	1·44	1·94	25	1·32	1·71
7	1 41	1·89	30	1·31	1·70
8	1·40	1·86	40	1·30	1·68
9	1·38	1·83	60	1·30	1·67
10	1·37	1·81	100	1·29	1·66
12	1·36	1.78	∞	1·28	1,64

Note — Linear interpolation for intermediate ν values is sufficiently accurate.

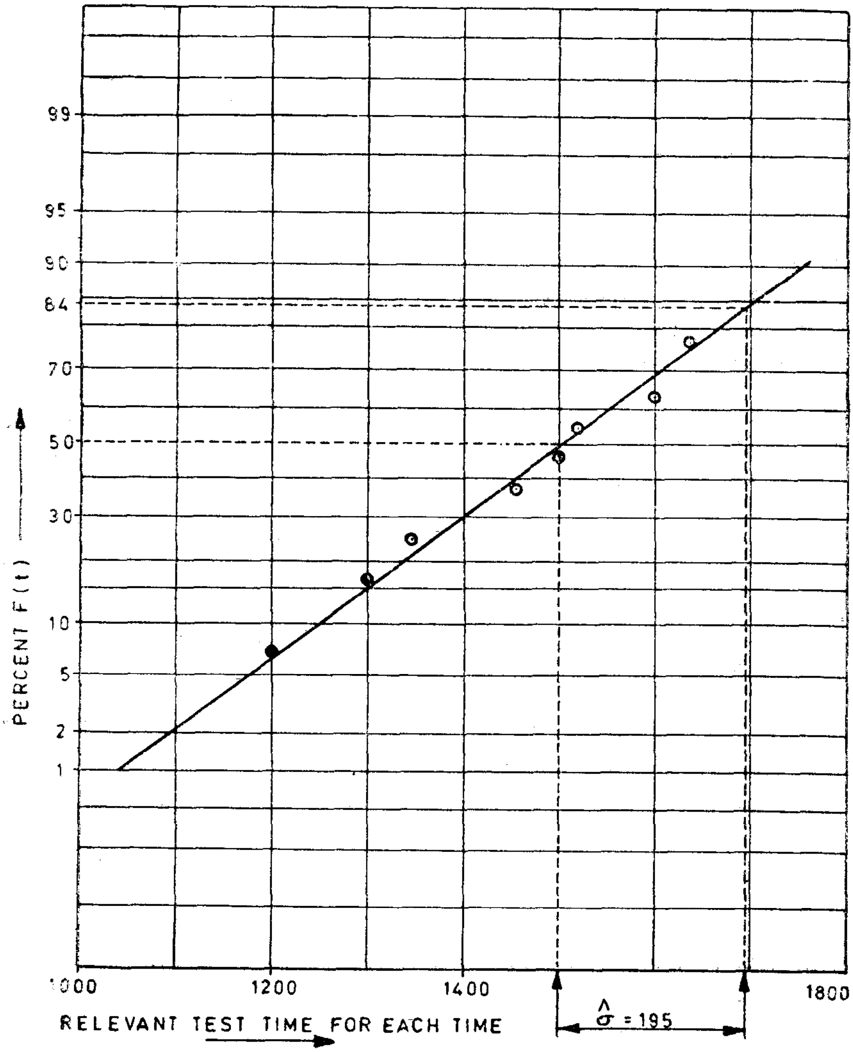
a) One-sided confidence interval, lower limit:

$$m_F > \hat{m}_F - t_{0\cdot90}\,(n-1)\,\frac{\hat{\sigma}}{\lambda_n}$$

b) Two-sided confidence interval:

$$\hat{m}_F - t_{0\cdot95}\,(n-1)\,\frac{\hat{\sigma}}{\lambda_n} < m_F < \hat{m}_F + t_{0\cdot95}\,(n-1)\,\frac{\hat{\sigma}}{\lambda n}$$

7.2.2 *Graphical Method*



In example, $n=10$, $r=8$

Failures occurred times in hours as plotted $m_F = 1\,500$

FIG. 5 EXAMPLE OF USE OF NORMAL PROBABILITY PAPER

7.2.2.0 The graphical method is based on the normal probability paper. This paper is so designed that the cumulative distribution function of times to failure, which follow a normal distribution, will become a straight line. *see* Fig. 5. It is applicable also in the case that not all items have failed.

7.2.2.1 The observed t_i values are plotted at the corresponding median rank values, $p_{50}(t_i)$, on the $F(t)$ scale. Median rank values may be found in Table 1 or approximated by the ratio $\frac{t - 0.3}{n + 0.4}$ where n is the number of test items. The example of Fig. 5 assumes a test sample $n = 10$ with the test terminated after $r = 8$ failures. In such a case, where the test is terminated before all items have failed, only the median ranks up to and including $i = r$ are plotted.

In drawing the line the points between the probabalities 20 percent and 80 percent should principally determine the line.

7.2.2.2 The estimate of the mean time to failure, m_F, is equal to the t value of the point on the straight line, which has an ordinate equal to 50percent.

7.2.2.3 The estimate of the standard deviation, $\hat{\sigma}$, is equal to the difference between the t value of the point on the straight line which has an ordinate equal to 84 percent and the estimated mean time to failure.

7.2.2.4 The fitted straight line may also be used to estimate the fraction of the population below or above a given time to failure.

8. SUCCESS RATIO

8.0 The success ratio is the probability that an item will perform a required function or that a trial will be successful under stated conditions.

When testing for success ratio, the test items or trials are to be classified as either failed or non-failed. Reusable devices may be repaired between successive trails, provided that the state and performance are the same at the start of all trials. The point estimate (observed success ratio) and the confidence interval for the success ration are independent of how the test is terminated at a fixed number of trials, a fixed number of successes or a fixed number of failures.

8.1 Point Estimate — The point estimate of success ratio is equal to the number of successes divided by the total number of trials or test items:

$$e = \frac{n - r}{n}$$

8.2 Confidence Intervals

8.2.0 The confidence limits for the true success ratio R with a confidence level of 90 percent, based on the binomial distribution, are

shown below. (*See* Tables 5 and 6 for values of the F distribution for 0·9 and 0·95 fractiles.)

TABLE 5 F DISTRIBUTION — 0·9 FRACTILES

$\nu_2 \diagdown \nu_1$	2	4	6	8	10	20	30	40	60	120	∞
2	9·00	9·24	9·33	9·37	9·39	9·44	9·46	9·47	9·47	9·48	9·49
4	4·32	4·11	4·01	3·95	3·92	3·84	3·82	3·80	3·79	3·78	3·96
6	3·46	3·18	3·05	2·98	2·94	2·84	2·80	2·78	2·76	2·74	2·72
8	3·11	2·81	2·67	2·59	2·54	2·42	2·38	2·36	2·34	2·32	2·29
10	2·92	2·61	2·46	2·38	2·32	2·20	2·16	2·13	2·11	2·08	2·06
12	2·81	2·48	2·33	2·24	2·19	2·06	2·01	1·99	1·96	1·93	1·90
14	2·73	2·39	2·24	2·15	2·10	1·96	1·91	1·89	1·86	1·83	1·80
16	2·67	2·33	2·18	2·09	2·03	1·89	1·84	1·81	1·78	1·75	1·72
18	2·62	2·29	2·13	2·04	1·98	1·84	1·78	1·75	1·72	1·69	1·66
20	2·59	2·25	2·09	2·00	1·94	1·79	1·74	1·71	1·68	1·64	1·61
30	2·49	2·14	1·98	1·88	1·82	1·67	1·61	1·57	1·54	1·50	1·46
40	2·44	2·09	1·93	1·83	1·76	1·61	1·54	1·51	1·47	1·42	1·38
60	2·39	2·04	1·87	1·77	1·71	1·54	1·48	1·44	1·40	1·35	1·29
120	2·35	1·99	1·82	1·72	1·65	1·48	1·41	1·37	1·32	1·26	1·19
∞	2·30	1·94	1·77	1·67	1·60	1·42	1·34	1·30	1·24	1·17	1·00

NOTE — Linear interpolation for intermediate ν values is sufficiently accurate.

TABLE 6 F DISTRIBUTION — 0·95 FRACTILES

$\nu_2 \diagdown \nu_1$	2	4	6	8	10	20	30	40	60	120	∞
2	19·00	19·20	19·30	19·40	19·40	19·40	19·50	19·50	19·50	19·50	19·50
4	6·94	6·39	6·16	6·04	5·96	5·80	5·75	5·72	5·69	5·66	5·63
6	5·14	4·53	4·28	4·15	4·06	3·87	3·81	3·77	3·74	3·70	3·67
8	4·46	3·84	3·58	3·44	3·35	3·15	3·08	3·04	3·01	2·97	2·93
10	4·10	3·48	3·22	3·07	2·98	2·77	2·70	2·66	2·62	2·58	2·54
12	3·89	3·26	3·00	2·85	2·75	2·54	2·47	2·43	2·38	2·34	2·30
14	3·74	3·11	2·85	2·70	2·60	2·39	2·31	2·27	2·22	2·18	2·13
16	3·63	3·01	2·74	2·59	2·49	2·28	2·19	2·15	2·11	2.06	2·01
18	3·55	2·93	2·66	2·51	2·41	2·19	2·11	2·06	2·02	1·97	1·92
20	3·49	2·87	2·60	2·45	2·35	2·12	2·04	1·99	1·95	1·90	1·84
30	3·32	2·69	2·42	2·27	2·16	1·93	1·84	1·79	1·74	1·68	1·62
40	3·23	2·61	2·34	2·18	2·08	1·84	1·74	1·69	1·64	1·58	1·51
60	3·15	2·53	2·25	2·10	1·99	1·75	1·65	1·59	1·53	1·47	1·39
120	3·07	2·45	2·18	2·02	1·91	1·66	1·41	1·37	1·32	1·26	1·19
∞	3·00	2·37	2·10	1·94	1·83	1·57	1·46	1·39	1·32	1·22	1·00

NOTE — Linear interpolation for intermediate ν values is sufficiently accurate.

8.2.1 One-sided confidence interval, lower limit:

$$R > \frac{n - r}{n - r + (r + 1) F_{90} (r_1, \nu_2)}$$

with $\nu_1 = 2 (r + 1)$ and $\nu_2 = 2 (n - r)$

8.2.2 Two-sided confidence interals:

$$\frac{n-r}{n-r+(r+1)F_{95}(\nu_1,\nu_2)} < R < \frac{(n-r+1)F_{95}(\nu_1,\nu_2)}{r+(n-r+1)F_{95}(\nu_1,\nu_2)}$$

with $\nu_1 = 2(r+1)$ and $\nu_2 = 2(n-r)$ at lower limits, and

$\nu_1 = 2(n-r+1)$ and $\nu_2 = 2r$ at upper limits

8.3 Use of Nomographs

8.3.1 Nomographs are presented for determining one-sided and two-sided confidence intervals of success ratio, *see* Fig. 6 and 7. The nomographs are based on the total number of test items or trials, n, and the total number of observed failures, r.

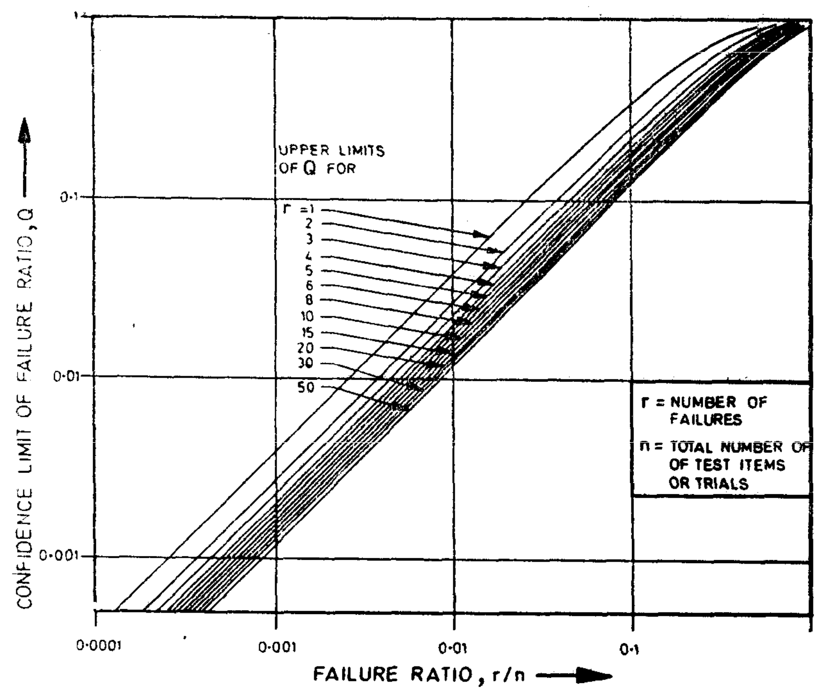


FIG. 6

8.3.2 Enter the applicable nomograph on the abscissa at the r/n value. Go vertically up to the curves labelled with the observed number of failures, r. Then go horizontally to the ordinate and to read the relevant failure ratio limits, Q. The confidence limits of the success ratio are the complements, that is, $R = 1 - Q$.

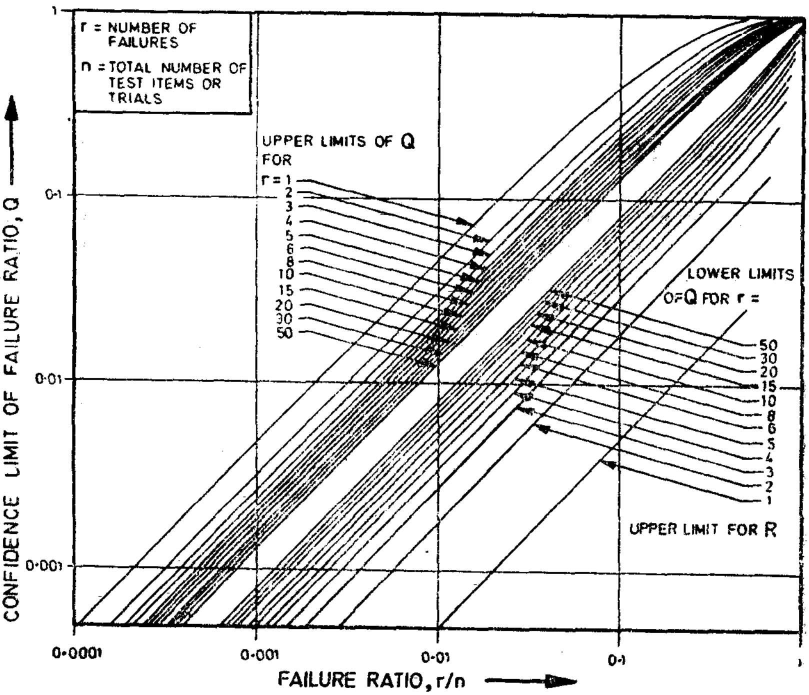

r = NUMBER OF FAILURES
n = TOTAL NUMBER OF TEST ITEMS OR TRIALS
UPPER LIMITS OF Q FOR
r = 1
2
3
4
5
6
8
10
15
20
30
50
LOWER LIMITS OF Q FOR r =
50
30
20
15
10
8
6
5
4
3
2
1
UPPER LIMIT FOR R
CONFIDENCE LIMIT OF FAILURE RATIO, Q
FAILURE RATIO, r/n
1
0·1
0·01
0·001
0·0001
0·001
0·01
0·1
1

FIG. 7

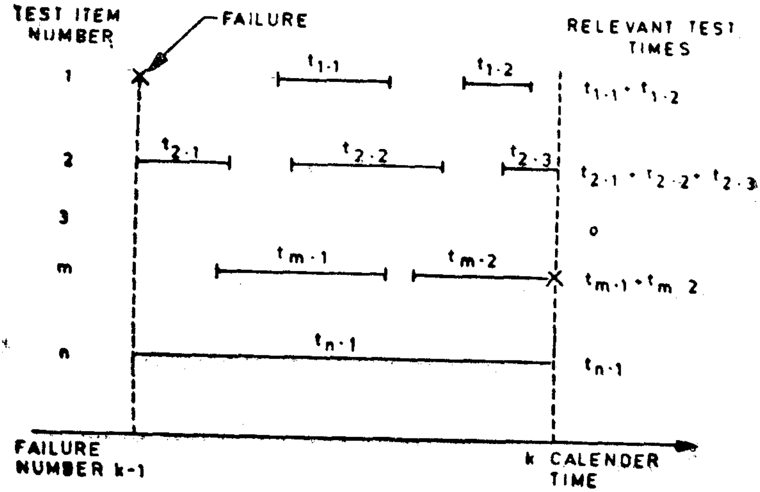

TEST ITEM NUMBER
FAILURE
RELEVANT TEST TIMES
1
2
3
m
n
t1·1
t1·2
t1·1 + t1·2
t2·1
t2·2
t2·3
t2·1 + t2·2 + t2·3
o
tm·1
tm·2
tm·1 + tm·2
tn·1
tn·1
FAILURE NUMBER k-1
k CALENDER TIME

FIG. 8

APPENDIX A

(*Clauses* 6.1.0 *and* 6.2)

DETERMINATION OF ACCUMULATED RELEVANT TEST TIME

A-1. The relevant test time of each individual test item can be measured by elapsed time indicators on the test item. In this case the accumulated relevant test time T_k at the kth failure is the sum of indicator readings.

$$T_k = \sum_{m=1}^{n} t_{k,\,m}$$

where

n = the total number of test items, and

$t_{k\,m}$ = the indicated relevant test item of number m up to the kth failure in the total number of test items.

A-2. The accumulated relevant test time T^* at a decision point not coinciding with a failure is:

$$T^* = \sum_{m=1}^{n} t_m^*$$

where

t_m^* = the indicated test time of item number m up to the decision point.

A-3. If the relevant test time is recorded by other means, the accumulated relevant test time T_k, at the kth failure should be calculated by the following iterative formula. This includes the accumulated relevant test time up to the failure $k - 1$ and the relevant test times elapsed in the interval between the failures $k - 1$ and k.

$$T_k = T_k - 1 + \sum_{m=1}^{n} \Sigma_J,\, t_{m,\,J}$$

where

n = the total number of test items, and

$t_{m,j}$ = the jth period of relevant test time of test item number m after the failure $k - 1$ in the total number of test items.

A-3. The intermissions may be caused by failure $k - 1$ or by any other technical or administrative reasons. The number, j, of intermissions may vary from item to item.

A-4. Figure 8 explains the numbering of periods of relevant test time.

A-5. The accumulated relevant test time T^* at a decision point not coinciding with a failure is in this case:

$$T^* = T_r + \sum_{m=1}^{n} \Sigma_j \, t_{m,j}$$

where

T_r = the accumulated relevant test time up to the latest failure before the decision point, and

$t_{m,j}$ = the jth period of relevant test time item number m after the latest failure in the total number of test items.

A-6. The above formulae are applicable also in case the test items are non-repaired equipments, except that relevant test time does not exist after the first failure of each test item.

INDIAN STANDARDS

ON

RELIABILITY OF ELECTRONIC AND ELECTRICAL COMPONENTS AND EQUIPMENT

IS:

1885 (Part 39)-1979 Electrotechnical vocabulary: Part 39 Reliability of electronic and electrical items (*first revision*)

2612-1965 Recommendation for type approval and sampling procedures for electronic components

7354 Guide on reliability of electronic and electrical items:

7354 (Part 1)-1975 Part 1 Preliminary reliability considerations

7354 (Part 2)-1975 Part 2 Managerial aspects of reliability

7354 (Part 3)-1975 Part 3 Presentation of reliability data on electronic and electrical components (or parts)

7354 (Part 4)-1974 Part 4 Collection of reliability, availability and maintainability data from field performance

7354 (Part 5)-1975 Part 5 Inclusion of lot-by-lot and periodic inspection procedures in specifications for electronic and electrical components (or parts)

7354 (Part 6)-1983 Part 6 Inclusion of reliability clauses into specifications for components (or parts) (*first revision*)

7690-1975 Mathematical guide to the terms and definitions for reliability of electronic equipment and components (or parts) used therein

8161 Guide for equipment reliability testings:

8161 (Part 1)-1976 Part 1 Principles and procedures

8161 (Part 5)-1981 Part 5 Compliance test plans for success ratio

8161 (Part 6)-1983 Part 6 Tests for validity of a constant failure rate assumption

8161 (Part 7)-1977 Part 7 Compliance test plans for failure rate and mean time between failures assuming constant failure rate

8161 (Part 11)-1983 Part 11 Flow chart describing preparations for and execution of reliability tests

9185 Endurance (life) test for electronic and electrical components:

9185 (Part 1)-1979 Part 1 Thermal endurance

9185 (Part 2)-1979 Part 2 Mechanical endurance

9186-1979 Guide for screening of electronic and electrical items

9692 Guide on maintainability of equipment:

9692 (Part 1)-1980 Part 1 Introduction to maintainability

9692 (Part 2)-1980 Part 2 Maintainability requirements in specifications and contracts

9692 (Part 3)-1981 Part 3 Maintainability programme

9692 (Part 5)-Part 5 Maintainability studies during the designphase

9692 (Part 6)-1983 Part 6 Maintainability verification

9692 (Part 7)-1984 Part 7 Collection, analysis and presentation of data related to maintainability

10139-1982 Presentation of reliability, maintainability and availability predictions

10673-1983 Sampling plans and procedures for inspection by attributes for electronic items

11137 (Part 2)-1984 Analysis techniques for system reliability: Part 2 Procedure for failure mode and effects analysis (EMEA) and failure mode, effects and criticality analysis (EMECA)